Horseshoe shaped conduit design

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Abstract. In practice, the main use of the horseshoe shaped ducts is as tunnels, to design such duct the diameter and other linear dimensions are indispensables, Moreover the horseshoe duct design requires using the uniform flow fundamental relations: Darcy-Weisbach, Colebrook-White and the Reynolds number. Unlike the common implicit method used to calculate horseshoe ducts, an explicit solution is proposed in this work to design the duct with simple and direct formulas.

Introduction

The three governing lows of the uniform flow through circular conduits (pipes) and noncircular conduits (ducts) are Darcy-Weisbach, Reynolds number and Colebrook-White. The relation of Darcy-Weisbach is used for the flow in conduits; within the friction factor f depends on the Reynolds number, the relative roughness and the cross section geometric parameters, it is very important in practice to express the friction factor of Darcy-Weisbach for the turbulent flow in conduits and channels. For the Colebrook-White equation [1] the friction factor f depends on Reynolds number and the relative roughness, where the Reynolds number is defined as the ratio of the inertia to viscous effects in the flow. These three relations found the basic for the turbulent pipe flow calculation. In general, the uniform flow encountered is correspond to values of Reynolds number upper than or equal to 2300 (because of the relatively small viscosity of most common fluids) and relative roughness between 0 and 0.05. Based on the Reynolds number three classifications of flow can be distinguished, which are laminar, transitional, and turbulent flow, this last often corresponds to somewhat higher values of relative roughness.

The three types of flow can be determined by the universal Moody diagram (Moody chart is a graphical representation of Colebrook formula, which is an empirical fit of the pipe flow pressure drop data). The principal objective of the Moody diagram is to determine the linear dimensions of pipes [3, 4, 5] and to contribute to study this type of conduits under pressure [6]. The uniform flow is characterized by the discharge, the slope of the energy line, the hydraulic radius (or diameter), the absolute roughness (irregularities in the surface of the pipe inner wall) and the kinematic viscosity.
According to Colebrook-White relation, the friction factor is implicit, besides that the geometric element of the channel is in an implicit form, the solution involves contains many tests and trying calculation [7]

Based on the theoretical approach “Rough Model Method” [8], the objective of this work is to calculate the linear dimensions of conduit, the advantage of this method that the resulting equations are explicit and cover the entire Moody diagram domain, corresponding to relative roughness range of 0 to 0.05 and Reynolds number bigger than 2300.

Geometric Elements of Horseshoe conduit

Fig.1 is a schematic representation of the horseshoe tunnel. It is characterized by the two geometric elements *y* and *r*, which represent the height of the bottom arc and the radius respectively, where $\theta = 0.294515\, rad$ and $R = 3r$.

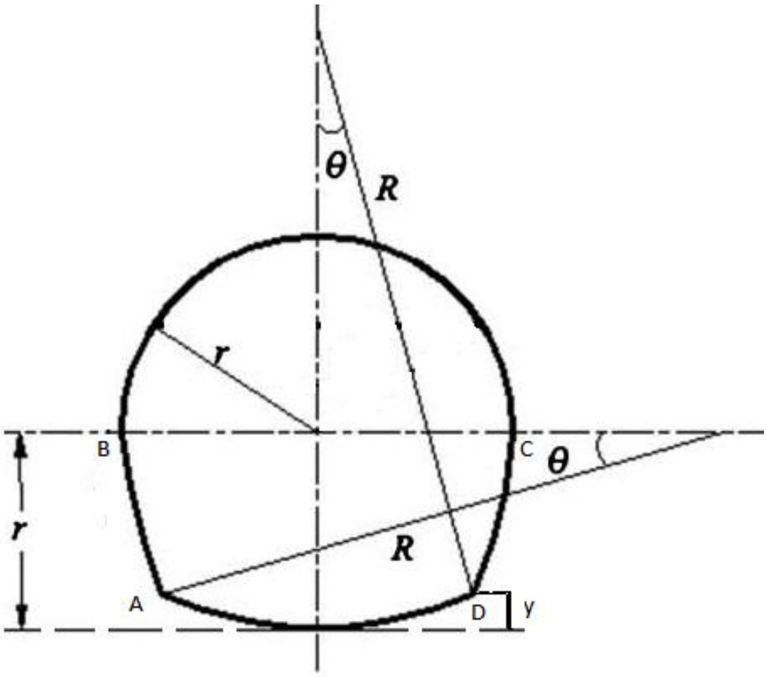

Fig.1. Geometric elements of the horseshoe conduit

The cross-sectional flow area *A* is expressed as:

$$A = 3{,}38875894 \times r^2 \tag{1}$$

And the wetted perimeter *P* is expressed as:

$$P = 6{,}6757726535\ 89 \times r \tag{2}$$

Thus the hydraulic diameter $D_h = 4\frac{A}{P}$ is:

$$D_h = 2{,}03048193 \times r \tag{3}$$

The Darcy-Weisbach relation as gives the energy slope J:

$$i = \frac{f}{D_h}\frac{Q^2}{2gA^2} \tag{4}$$

Where *Q* is the discharge, *g* is the gravitational acceleration and *f* is the friction factor given by the well-known Colebrook-White formula as:

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\varepsilon / D_h}{3{,}7} + \frac{2{,}51}{R\sqrt{f}}\right) \tag{5}$$

ε is the absolute roughness and *R* is the Reynolds number which can be expressed as :

$$R = \frac{4Q}{P\nu} \tag{6}$$

Where ν is the kinematic viscosity.

Uniform Flow Computation

Both of geometric and hydraulic characteristics of the rough model are distinguished by the symbol "$\bar{}$". The rough model we consider is a horseshoe conduit characterized by $\bar{\varepsilon}/\bar{D}_h = 0.037$ as an arbitrarily assigned relative roughness value. The chosen relative roughness value is so large that the prevailed flow regime is fully turbulent. Thus, the friction factor is f =1/16 according to Eq. 6 for $R = \bar{R}$ tending to infinitely large value. Applying Eq.5 to the rough model leads to:

$$\bar{i} = \frac{\bar{f}}{\bar{D}_h}\frac{\bar{Q}^2}{2g\bar{A}^2} \quad (7)$$

Eq. 7 is rewritten as:

$$\bar{i} = \frac{1}{128g}\frac{\bar{P}}{\bar{A}^3}\bar{Q}^2 \quad (8)$$

Introducing Eq. (1) and Eq. (3) into Eq. 9, one may write:

$$i = 0{,}00134019\ 977\left(\frac{Q^2}{g\bar{r}^{-5}}\right) \quad (9)$$

Let us assume $\bar{Q} = Q$, $\bar{i}$=i and $\bar{r} \neq r$. One can deduce from Eq.9:

$$\bar{r} = 0{,}26633847\ 44\left(\frac{Q}{\sqrt{gi}}\right)^{2/5} \quad (10)$$

The wetted perimeter of the rough model is given as:

$$\bar{P} = 1{,}7780151\left(\frac{Q}{\sqrt{gi}}\right)^{2/5} \quad (11)$$

Introducing Eq.10 in Eq. 6 becomes:

$$\bar{R} = 2{,}24969967\ 3\frac{(gQ^3 i)^{1/5}}{\nu} \quad (12)$$

Correction Factor of Linear Dimension

The RMM states that the linear dimension r of the conduit and that of the rough model $\bar{r}$ are related by the following equation:

$$r = \psi\bar{r} \quad (13)$$

Where Ψ is a non-dimensional correction factor for the linear dimension, this correction factor is less than one and the following relation [8] could calculate it:

$$\psi \cong 1{,}35\left[-\log\left(\frac{\varepsilon/\bar{D}_h}{4{,}75} + \frac{8{,}5}{\bar{R}}\right)\right]^{-2/5} \quad (14)$$

Computation Steps of the Linear Dimension

Knowing the flow rate Q, the energy slope i, the absolute roughness ε and the kinematic viscosity ν, the following sequence leads to calculate the required radius r and the linear dimensions:

1. Knowing Q, i, calculate the linear dimension $\bar{r}$ of the rough model from Eq. 10.
2. Compute the wetted $\bar{P}$ perimeter and the hydraulic diameter $\bar{D_h}$ of the rough model using the Eq. 11 and Eq. 3 respectively.
3. Knowing Q, $\bar{P}$ and ν , compute the Reynolds number $\bar{R}$ of the rough model using Eq. 12.
4. Applying Eq. 14, the dimensionless correction factor Ψ is then calculated.
5. The required value of the linear dimensions finally could be given as:

. r is $r = \psi \bar{r}$ according to Eq. 13.
.arc $AB = CD = 0.883545 \times r$
.arc $AD = 1.76709 \times r$
. $y = 0.1291708841\ 17 \times r$

Example

Calculate the linear dimensions of the horseshoe conduit, using the following DATA:
Q =3.218m^3/s, i =0.0004, ε=0.001m, ν=0.000001m^2/s
1. According to the Eq.10 the radius of the rough model is:

$$\bar{r} = 0{,}26633847\left(\frac{3.218}{\sqrt{9.81 \times 10^{-3}}}\right)^{2/5} = 1{,}28742674 \text{ m}$$

2. The wetted perimeter and the hydraulic diameter of the rough model are:

$$\bar{P} = 1{,}778015\left(\frac{3.218}{\sqrt{9.81 \times 10^{-3}}}\right)^{2/5} = 8{,}5945682 \text{ m}$$

$$\bar{D}_h = 2{,}03048193 \times 1{,}287426 = 2{,}61409673\, m$$

3. According to Eq.12 the Reynolds number of the rough model is:

$$\bar{R} = 2{,}24969967\ 3\frac{(gQ^3 i)^{1/5}}{\nu} = 1497690{,}13$$

4. According to Eq.14, the non-dimensional correction factor Ψ is then:

$$\psi = 1.35\left[-\log\left(\frac{0.001/2{,}61409}{4.75} + \frac{8.5}{1497690{,}13}\right)\right]^{-2/5} = 0{,}77043055\ 1$$

8. Finally, the required value of the linear dimensions:
$r = 0{,}77004305\ 51 \times 1{,}28742674 = 0{,}99187283$ m
$y = 0{,}1291708841\ 17 \times 0{,}99187 = 0{,}12812109$ m
Arc $AB = CD = 0{,}883435 \times 0{,}991872 = 0{,}87636428$ m
Arc $AD = 1{,}767009 \times 0{,}9918 = 1{,}75272857$ m

Conclusion

The main objective of this work is to give simple sequence of steps, based on an analytical method, in order simplify the design of the horseshoe shaped conduit. The proposed method consists of explicit relations.

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