The effective length estimation of columns in semi-rigid jointed braced frames

Ismail Meghezzi-Larafi[✉], **Abdelouahab Tati**

Laboratoire de Génie Energétique et Matériaux, LGEM, Université de Biskra, BP145 Biskra 07000, Algeria

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Abstract: This paper presents a theoretical and exact procedure for the stability analysis of braced steel frames, taking in account the flexibility effect of the beam-column connections. In order to determine the effective length factor (K-factor). The isolated subassembly approach is used to establish the buckling governing equation. In this study, the relative stiffness coefficient at the isolated column ends is provided by the remainder members of the structure, rather than of the relative stiffness factor in the alignment chart method. A computer program for plane structure analysis is used to evaluate the relative stiffness coefficients. To illustrate the accuracy of the established transcendental equation, K-factor values for the case of fully rigid connections, are compared to the exact and the French rules results. The effect of the type of transfer elements between the frame members, in term of fixity factors is investigated. Moreover, the effect of restraining conditions, provided by the whole frame structure, in term of relative stiffness coefficients, is also studied. The obtained results revealed that the buckling critical loads of the columns in frames of rigid members are significantly affected by the fixity factors variation, unlike in flexible structures.

1. Introduction

For decades, researchers have developed various approaches for assessing column and frame stability in the design of steel structures. Most approaches, known as the effective length method, deal with the effective length factor (herein K-factor) and the buckling strength of individual compressed members. Aimed at providing sufficient simplicity for hand calculations, the effective length method is based on some assumptions that may have considerable influence on accuracy (Gantes and Mageirou 2005).

In current design practice, the connections of beam-to-column and column bases in steel frames are often idealized either as purely pinned or fully rigid connections. However, numerous experimental researches showed for a long time, that the behavior of these latter is rather between these two extremes (Picard and Beaulieu 1985; Canadian CSA 1989; Montforton and Wu 1963). The semi-rigid behavior of beam-columns connections of the structures, affects the internal efforts in the structure members and consequently, the buckling critical loads and corresponding effective lengths of the compressed elements (Simoes 1996; Webber et al. 2015).

It is well known that the maximum strength of frames and the maximum strength of an axially loaded column are interrelated. Therefore the determination of critical buckling loads of columns in braced or unbraced steel frame columns is of primordial importance during design. Various methods have been proposed for the analysis of the frames stability and for the evaluation of the effective length factor of columns. All these methods get into three approaches, namely, the isolated subassembly approach, the story-based- approach and the system buckling approach. Al these methods consider the assumption of considering only the effect of beams and columns, connected directly to the considered column G-factor (Bridge and Fraser 1987), rather than the rigidity effect, provided by the overall structure.

Since the 60s, the evaluation of the *K*-factor for semi-rigid jointed braced frames has been considered by many researchers (Barakat and Chen 1990; Dumonteil 1992; Kavanagh 1962; Tong and Wang 2004). Kishi et al. 1998, who proposed a procedure to evaluate the effective length factor of columns in flexibly jointed and braced frames, based on classical alignment chart (Goto et al 1993). The authors introduced the modified relative stiffness factor to account for the non-linear connection stiffness. Their study shows that the alignment chart can be applied to determine K-factor for flexibly jointed and rigid frames by estimating the tangent connection stiffness at the buckling (Xu and Liu 2002) presented a practical method for the stability analysis of semi-braced steel frames, based on the concept of story-based buckling. They introduced the concept of the lateral bracing factor to characterize the lateral bracing for the column and frame. The author found that the critical buckling loads of the frame increase considerably as the end-fixity factors increase from zero to 0.3 for beam-to-column connections. (Tong and Wang 2004) proposed a new method for determining the column effective length in one-span symmetrical frames; its high accuracy has been displayed through a large number of

Corresponding author. E-mail address: ksmail7176@gmail.com

examples. Xu and Liu (2002) extend the proposed method to multi-span multi-story unsymmetrical frames in which they considered both the inter-column and the inter-story interaction. The authors have proposed a method of evaluating the critical frame-buckling loads for multi-story unbraced steel frames subjected to variable loading. The validity of the proposed method is demonstrated by two-bay two stories unbraced steel frames. The authors have considered the effects of the connection behavior on the critical frame-buckling loads in a variable case, as well as cases with different rigidities of beam to-column connections (Webber et al. 2015). Recently, Hellesland (2012) proposed approximate formulas for effective length evaluation of isolate compressed members, where positive and negative restraints are reviewed and discussed.

Based on the derivation of the elastic stability equation for a braced column, a method of evaluating the buckling load and *K*-factors for braced frames is presented in this paper. A procedure based on the global effect of the frame on the compressed member in multistory braced and semi rigid jointed frames is also presented. The effect of the frame on the ends of the column is modeled by rotational springs whose rigidities are obtained by applying unity pointed moments and computing the rotational angles at points, corresponding to the ends of the considered column. Calculating the rigidity is achieved using a FORTRAN program for computing planar structures. This calculation code considers the semi-rigid connections between the members of the structure. The buckling equations are derived based on nonlinear moment rotation relationship taking into account the proper stiffness of the connection in term of fixity factor. The obtained transcendental equation is solved for different restraining rigidities and fixities factors. Numerical examples have demonstrated that the proposed method is efficient to estimate the buckling load and corresponding *K*-factors for the braced frames.

2. Modeling of isolated member

In this study based on the isolated member analysis, the interaction between the column AB and the remainder of the structure, can be reflected by the springs restraints of stiffness's C_A and C_B, as illustrated in Figure 1, while the effect of the flexible transfer elements at the ends is represented by two other springs having λ_A and λ_B as coefficients of flexibility.

3. Moment–rotation relationship of a column

According to the semi rigid column shown in Figure 2, the rotations angles at the column ends A and B, considering large displacement and proper connections rotations are given by (Timoshenko and Gere 1966):

$$\theta_A - \alpha_A = \frac{M_A L_c}{3EI_c}\psi + \frac{M_B L_c}{6EI_c}\varphi \tag{1a}$$

$$\theta_B - \alpha_B = \frac{M_{AL_c}}{6EI_c}\varphi + \frac{M_B L_c}{3EI_c}\psi \tag{1b}$$

Where α_A and α_B are proper connections rotations at ends A and B respectively.

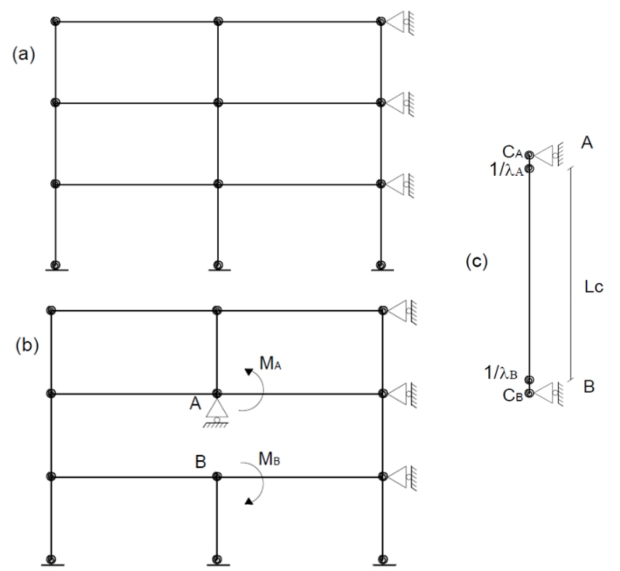

Fig.1. Braced frame (a), Remainder frame (b) and isolated member (c)

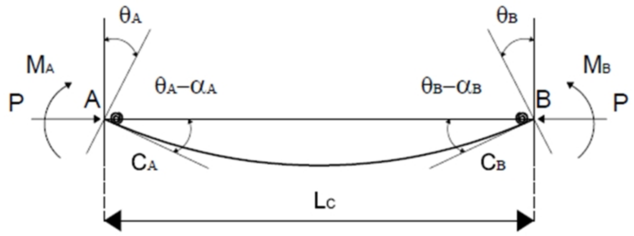

Fig.2. Isolated member modeling.

Where α_A and α_B are proper connection rotations at the ends A and B, respectively;

E: Young's modulus;

I_c: moment of inertia of the column;

L_c: column length;

P: compression load;

φ and ψ are functions of P, L_c , E and I_c

$$\varphi = \frac{3}{u}\left(\frac{1}{\sin 2u} - \frac{1}{2u}\right) \tag{2a}$$

$$\psi = \frac{3}{2u}\left(\frac{1}{2u} - \frac{1}{\tan 2u}\right) \tag{2b}$$

Where

$$u = \frac{L_c}{2}\sqrt{\frac{P}{EI_c}} \tag{3}$$

The proper rotations of the connections A and B are related to the ends moments M_A and M_B by:

$$\alpha_A = \lambda_A M_A \tag{4a}$$

$$\alpha_B = \lambda_B M_B \tag{4b}$$

Where λ_A and λ_B are the proper flexibilities of the connections A and B respectively.

λ_A and λ_B are related to fixity factors of the connections A and B by Montforton and Wu (1963):

$$\gamma_A = \frac{L_c}{L_c + 3EI_c\lambda_A} \tag{5a}$$

$$\gamma_B = \frac{L_c}{L_c + 3EI_c\lambda_B} \tag{5b}$$

For a column with rigid ends, the corresponding values of γ_A and γ_B are unity because the values λ_A and λ_B are taken to be zero.

For a column with purely pinned ends, the connections flexibilities λ_A and λ_B are to be infinite and the corresponding values of γ_A and γ_B are zero.

The values of γ_A and γ_B are between zero and unity for a column with semi-rigid ends conditions.

The fixity factor value is experimentally determined and it depends on the transfer element type, used to assemble different members of the frame structure (Picard and Beaulieu 1985).

Then, the upper and lower connections flexibilities are related to corresponding fixity factors by:

$$\lambda_A = \frac{L_c}{3EI_c}\frac{1-\gamma_A}{\gamma_A} \tag{6a}$$

$$\lambda_B = \frac{L_c}{3EI_c}\frac{1-\gamma_B}{\gamma_B} \tag{6b}$$

Finally the equations (1 a) and (1 b) can be expressed in terms of the ends fixity factors as:

$$\theta_A = \frac{M_A L_c}{3EI_c}\left(\psi + \frac{1-\gamma_A}{\gamma_A}\right) + \frac{M_B L_c}{6EI_c}\varphi \tag{7a}$$

$$\theta_B = \frac{M_{AL_c}}{6EI_c}\varphi + \frac{M_B L_c}{3EI_c}\left(\psi + \frac{1-\gamma_B}{\gamma_B}\right) \tag{7b}$$

4. Derivation of K-factor governing equation of the column with semi-rigid ends conditions

The rotations θ_A and θ_B of the connections A and B of the frame (Figure. 1b) are related to the applied unite torques M_A and M_B by the stiffness coefficients C_A and C_B , provided by the hole frame structure

$$C_A = \frac{M_A}{\theta_A} \tag{8a}$$

$$C_B = \frac{M_B}{\theta_B} \tag{8b}$$

C_A and C_B are determined by applying unite torques (M_A = M_B = 1) at the upper and lower connections A and B of the frame structure (Figure. 1b).

By substituting (Eq. 8) in (Eq. 7) we can obtain the following equations:

$$M_A\left(\frac{1}{C_A} + \frac{L_c}{3EI_c}\left(\psi + \frac{1-\gamma_A}{\gamma_A}\right)\right) + M_B\frac{L_c}{6EI_c}\varphi = 0 \tag{9a}$$

$$M_A\frac{L_c}{6EI_c}\varphi + M_B\left(\frac{1}{C_B} + \frac{L_c}{3EI_c}\left(\psi + \frac{1-\gamma_B}{\gamma_B}\right)\right) = 0 \tag{9b}$$

Eqs. (9a) and (9b) can be written in the following matrix form:

$$\begin{bmatrix} \frac{1}{C_A} + \frac{L_c}{3EI_c}\left(\psi + \frac{1-\gamma_A}{\gamma_A}\right) & \frac{L_c}{6EI_c}\varphi \\ \frac{L_c}{6EI_c}\varphi & \frac{1}{C_B} + \frac{L_c}{3EI_c}\left(\psi + \frac{1-\gamma_B}{\gamma_B}\right) \end{bmatrix} \begin{Bmatrix} M_A \\ M_B \end{Bmatrix} = 0 \tag{10}$$

This equations system can have two possible solutions:

The first is the banal solution of M_A = M_B = 0, in this case buckling do not occur.

The second solution which characterizes the buckling state is by setting:

$$\det\begin{bmatrix} \frac{1}{C_A} + \frac{L_c}{3EI_c}\left(\psi + \frac{1-\gamma_A}{\gamma_A}\right) & \frac{L_c}{6EI_c}\varphi \\ \frac{L_c}{6EI_c}\varphi & \frac{1}{C_B} + \frac{L_c}{3EI_c}\left(\psi + \frac{1-\gamma_B}{\gamma_B}\right) \end{bmatrix} = 0 \tag{11}$$

The Euler buckling load of the column is given by:

$$P_e = \frac{\pi^2 EI_c}{L_c^2} \tag{12}$$

The buckling critical load of the column is given by:

$$P_{cr} = \frac{4u^2 EI_c}{L_c^2} = \frac{\pi^2 EI_c}{(KL_c)^2} \tag{13}$$

In which, K denote the effective length factor.

by setting:

$$R_A = \frac{EI_c}{L_c C_A} \tag{14a}$$

$$R_B = \frac{EI_c}{L_c C_B} \tag{14b}$$

Where $\frac{EI_c}{L_c}$ denote the flexural stiffness of the column.

The general K-factor equation can be expressed as follows:

$$F_1 + F_2 + F_3 + F_4\frac{3K}{\pi}\left(\frac{K}{\pi} - \frac{1}{\tan\frac{K}{\pi}}\right) + \frac{\pi^3}{8K^3}\left(\tan\frac{\pi}{2K} - \frac{\pi}{2K}\right) = 0 \tag{15}$$

In which:

$$F_1 = 12\ R_A\gamma_A(1-\gamma_B) + 12\ R_B\gamma_B(1-\gamma_A)$$

$$F_2 = 4(1-\gamma_A)(1-\gamma_B)$$

$$F_3 = 36\ R_A R_B \gamma_A \gamma_B$$

$$F_4 = 4(\gamma_A + \gamma_B - 2\gamma_A\gamma_B)$$

In practice, the connections in a frame structure are identical and have almost the same behavior, and as a result the same fixity factors. Therefore in this particular case we can consider that $\gamma_A = \gamma_B = \gamma$. Thus the K-factor governing equation can be reduced to:

$$G_1 + G_2 + G_3 + G_4 \frac{3K}{\pi}\left(\frac{K}{\pi} - \frac{1}{\tan\frac{K}{\pi}}\right) + \frac{\pi^3}{8K^3}\left(\tan\frac{\pi}{2K} - \frac{\pi}{2K}\right) = 0 \quad (16)$$

In which:

$$G_1 = 12(R_A + R_B)\gamma(1-\gamma)$$

$$G_2 = 4(1-\gamma^2)$$

$$G_3 = 36 R_A R_B \gamma^2$$

$$G_4 = 8\gamma(1-\gamma)$$

In the case of column pinned at the low end and partially fixed at the upper end, the moment at the lower end is equal to zero. The K-factor governing equation can be reduced to the following expression:

$$\frac{3K}{\pi}\left(\frac{K}{\pi} - \frac{1}{\tan\frac{K}{\pi}}\right) + 3R_B = 0 \quad (17)$$

5. K-factor of columns in braced frame structure

5.1 Results comparison

In order to illustrate the accuracy of the proposed procedure, a comparative study is conducted using a few sample points. Table 1 show the comparison of the effective length factor K obtained by solving Eq. (16) of the present work, and that obtained by the approximate French Rules (Dumonteil 1992) with those obtained by solving the corresponding exact equations (Barakat and Chen 1990) for braced frames and sway frames in the case of rigidly fixed connections (γ=1), respectively. The results show the accuracy of the derived equations compared to the exact results. For these reasons, we can consider the obtained results as exact.

5.2 K-factor of columns flexibly jointed and braced frames

In order to evaluate the effective length factor values for different restraining conditions and different fixity factors values of a column situated at any story of the multi-story braced frames, the transcendental Eq. (16) and Eq. (17) are solved, using a numerical iterative method. From mathematical point of view, the results are supposed to be exact taking into account physical assumptions; the decision should be taken by the designer. The obtained results of the effective length factor (K-factor) are presented in tables (2, 3, and 4). The effective length factor has been evaluated for a large range of stiffness coefficients R_A and R_B and for three values of fixity factors (γ = 1, γ = 0.6 and γ = 0.3). The results show that for R_A = R_B = 10, the column has substantially the same behavior of a pinned column.

5.3 K-factor of columns flexibly jointed and braced frames

The curves of (Figure 3) show the variation of K-factor versus the ends relative rigidity coefficients (R = R_A = R_B) of a columns in braced frames for different values of the fixity factor (γ). According to the curves K-factors converge towards unity for relative rigidity coefficients R >3.0. The frame columns behave as pinned ends columns, of critical buckling load equal to Euler's critical load, therefore the fixity factor has no effect on connections relative rigidity coefficients greater than 3.0. It can also be seen according to the curves, that for fixity factor values higher than 0.6 the values of K-factor are closer to each other . However for γ lower than 0.4 the behavior of the connection is too close to that of a pinned ends column.

Table 1. Effective length factor (K) comparison (Fully fixed).

G_A	G_B	R_A	R_B	Present work	Exact (Hellesland 2012)	French rule (Dumonteil 1992)
0.10	0.400	0.050	0.200	0.6030	0.603	0.608
0.25	0.250	0.125	0.125	0.6110	0.611	0.619
0.10	0.900	0.050	0.450	0.6480	0.648	0.651
0.25	0.750	0.125	0.375	0.6717	0.672	0.677
0.50	0.500	0.250	0.250	0.6862	0.686	0.692
0.10	1.900	0.050	0.950	0.6829	0.683	0.685
0.25	1.750	0.125	0.875	0.7158	0.716	0.721
0.50	1.500	0.250	0.750	0.7510	0.751	0.756
1.00	1.000	0.500	0.500	0.7742	0.774	0.778
0.50	4.500	0.250	2.250	0.7923	0.792	0.798
1.00	4.000	0.500	2.000	0.8402	0.840	0.844
2.50	2.500	1.250	1.250	0.8772	0.877	0.879
0.50	9.500	0.250	4.750	0.8064	0.806	0.813
1.00	9.000	0.500	4.500	0.8583	0.858	0.862
2.50	7.500	1.250	3.750	0.9129	0.913	0.914
5.00	5.000	2.500	2.500	0.9302	0.930	0.931
50.0	4.000	25.00	2.000	0.9524	0.952	0.953
50.0	10.00	25.00	5.000	0.9770	0.977	0.977
100.0	50.00	50.00	25.00	0.9940	0.994	0.994

Table 2. K-factor for fully fixed and braced column γ = 1.

R_A \ R_B	0.0	0.25	0.50	0.75	1.00	2.00	5.00	10.00
0.00	0.5000	0.5895	0.6260	0.6445	0.6555	0.6750	0.6889	0.6939
0.25	0.5895	0.6863	0.7287	0.7510	0.7647	0.7892	0.8070	0.8136
0.50	0.6260	0.7287	0.7743	0.7982	0.8133	0.8402	0.8599	0.8672
0.75	0.6445	0.7510	0.7982	0.8237	0.8392	0.8675	0.8884	0.8961
1.00	0.6555	0.7647	0.8133	0.8392	0.8553	0.8846	0.9061	0.9140
2.00	0.6750	0.7892	0.8402	0.8676	0.8846	0.9156	0.9385	0.9470
5.00	0.6889	0.8070	0.8599	0.8884	0.9061	0.9385	0.9625	0.9714
10.00	0.6939	0.8136	0.8672	0.8961	0.9140	0.9470	0.9714	0.9805

Table 3. K-factor for semi-rigid fixed ends and braced column γ = 0.6.

R_A \ R_B	0.0	0.25	0.50	0.75	1.00	2.00	5.00	10.00
0.00	0.6721	0.7178	0.7413	0.7554	0.7648	0.7836	0.7989	0.8049
0.25	0.7178	0.7670	0.7925	0.8080	0.8184	0.8392	0.8563	0.8630
0.50	0.7413	0.7925	0.8193	0.8355	0.8465	0.8685	0.8865	0.8937
0.75	0.7554	0.8080	0.8355	0.8523	0.8636	0.8864	0.9051	0.9126
1.00	0.7648	0.8184	0.8465	0.8636	0.8751	0.8984	0.9176	0.9253
2.00	0.7836	0.8392	0.8685	0.8864	0.8984	0.9228	0.9430	0.9510
5.00	0.7989	0.8563	0.8865	0.9051	0.9176	0.9430	0.9640	0.9640
10.00	0.8049	0.8630	0.8937	0.9126	0.9253	0.9510	0.9724	0.9809

Table 4. Factor for semi-rigid fixed ends and braced column γ = 0.3.

R_A \ R_B	0.0	0.25	0.50	0.75	1.00	2.00	5.00	10.00
0.00	0.8278	0.8428	0.8529	0.8603	0.8659	0.8792	0.8928	0.8991
0.25	0.8428	0.8581	0.8686	0.8762	0.8820	0.8957	0.9097	0.9162
0.50	0.8529	0.8686	0.8793	0.8871	0.8930	0.9070	0.9213	0.9280
0.75	0.8603	0.8762	0.8871	0.8950	0.9010	0.9152	0.9298	0.9366
1.00	0.8659	0.8820	0.8930	0.9010	0.9070	0.9215	0.9362	0.9431
2.00	0.8792	0.8957	0.9070	0.9152	0.9215	0.9363	0.9515	0.9586
5.00	0.8928	0.9097	0.9213	0.9298	0.9362	0.9515	0.9672	0.9745
10.00	0.8991	0.9162	0.9280	0.9366	0.9431	0.9586	0.9745	0.9819

5.4 Fixity factor effect

In practice the connections beam-column in frame structures are idealized and are considered as fully rigid or pinned. However the actual behavior of this connection is between these extremes. In order to know the effect of partially rigid connections on the buckling critical load of a column in braced frames, a parametric study is done for different values of connections relative rigidity coefficient R.

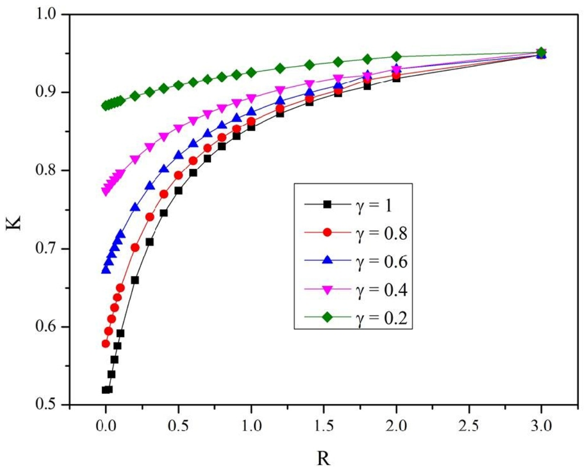

Fig.3. Braced effective length factor versus relative rigidity coefficient.

Curves of Figure 4 show the variation of the K-factors versus the fixity factor for different values of R.

6. Numerical example

In the first example, three-storey steel frame, as shown in Figure 5, is considered. The moments of inertia for the beam and columns are $I_b = 2I$ and $I_c = I$, respectively. Young's modulus of steel is taken as E = 200 GPa.

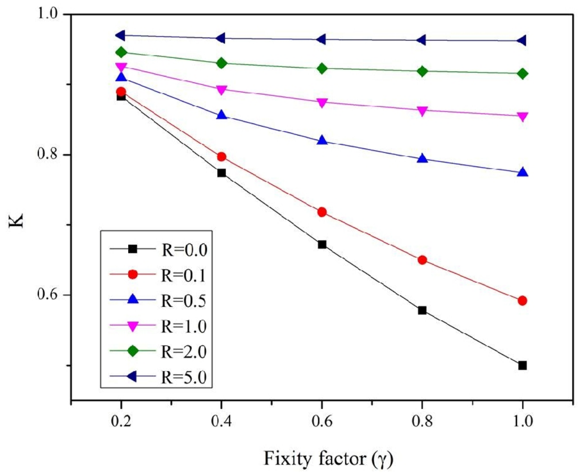

Fig.4. Braced effective length factor versus fixity factor (γ).

The frame has semi-rigid connections of equal fixity factors at the two ends of each of the members. Fixity factors values considered are γ = 1.0, γ = 0,6 and γ = 0.3 The process of evaluation of the effective length factors of the median frame columns is shown below:

1- Step 1: computing the rigidities C_A and C_B of the lower and upper connections A and B respectively by calculating θ_A and θ_B (Figure 5) for the considered story. To do so, the plane structures calculation program is used.

2- Step 2: calculation of the relative stiffness coefficients R_A and R_B by mean of Eq. (14).

Solve the transcendental Eqs. (16) or (17) to evaluate the effective length factor of a column in a semi-rigid jointed frame for a given value of fixity factor γ.

Table 5 summarizes the effective length factors for the inner columns of the frame found using the proposed method. The effective length factors for the columns in each story are also illustrated in Figure.6. For this frame, the weakest columns are in the upper story because the relative rigidities provided to their connections are less than those of the lowest columns. We can also see, according to the Figure 6 that the three curves are almost identical and have the same allure. According to Figure 7, it could be shown that the K-factor of the column in a story has a very linearly versus the fixity factor and in the same way.

Table 5. Numerical example

Fixity factor γ	Story	R_A	R_B	K
1.0	1	0.09147	0	0.5422
	2	0.09366	0.08966	0.5852
	3	0.14671	0.09367	0.6065
0.6	1	0.1739092	0	0.7073
	2	0.174775	0.17256	0.7444
	3	0.266062	0.17455	0.7578
0.3	1	0.364975	0	0.8478
	2	0.365275	0.36412	0.8687
	3	0.549540	0.36500	0.8757

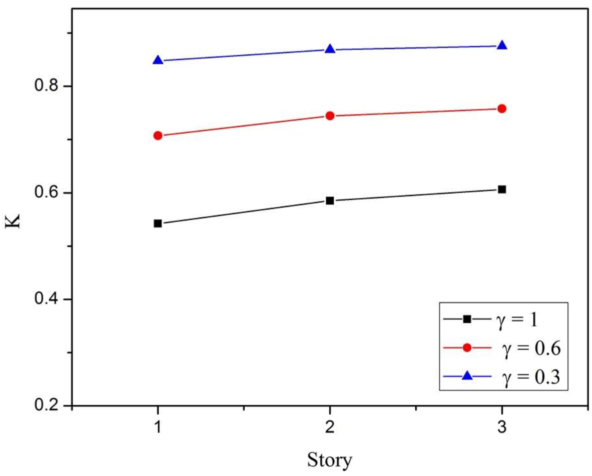

Fig.6. K-factor of stories columns.

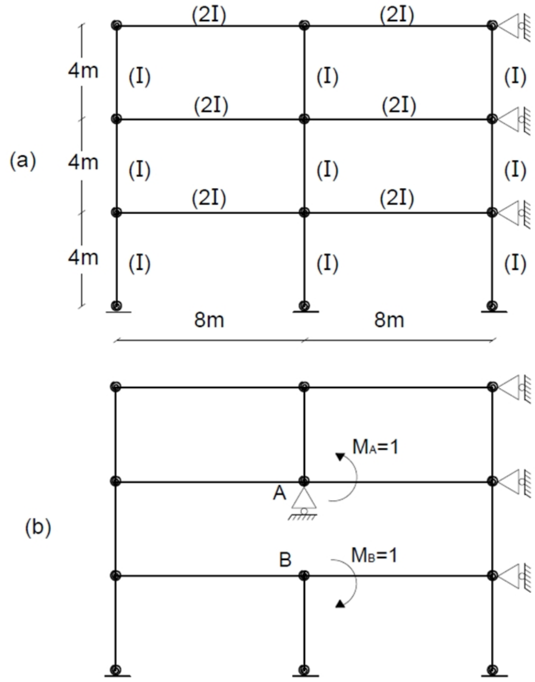

Fig.5. Numerical example (a) Braced frame, (b) Remainder frame.

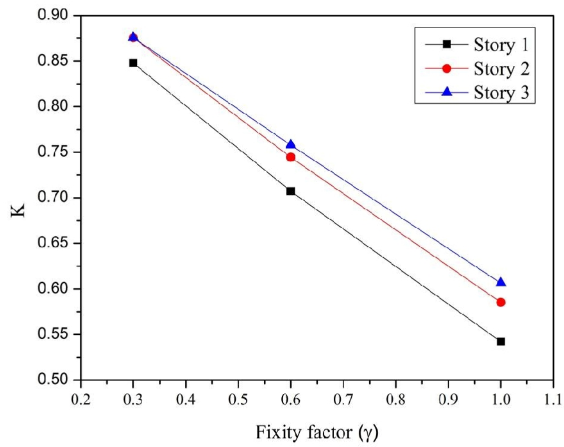

Fig.7. Numerical example (K-factor vs γ).

7. Conclusions

In current engineering practice, the evaluation of the K-factor is essential for the rational design of semi-rigid jointed frames. In this paper, the governing equation for determining the column *K*-factor for flexibly jointed and braced frames under various boundary conditions using the slope–deflection equation approach is derived on the basis of the strength of material. It was shown, according to the comparative study that the developed equation is mathematically exact and can be used for a parametric study. Indeed, the relative rigidity coefficient of the column ends provided by the remainder of the structure for different values of fixity factor has been studied and the results show that after the value of 0.3, the K-factor reach the unity and the column has almost the behavior of the ends pinned column for any fixity factor value.

In this paper the beam-columns connections flexibilities effect have been studied in term of fixity factors and the results showed that the effect of γ is very significant for rigid structures ($R < 0.5$). However the effect of the fixity factor has almost no effect for flexible frame structures and K-factor is almost constant, especially for the values of R greater than 5. Also a numerical example is dealt with in this paper. The results show that the lowest columns are more stable for any type of transfer element illustrated in term of fixity factors. For fully rigid jointed frame ($\gamma = 1$), the critical buckling load of the lowest column is 1.25 time of the upper one. However for semi-rigid jointed frame, the ratio between the critical buckling loads of the lowest and the upper columns is respectively 1.15 for $\gamma = 0.6$ and 1.07 for $\gamma = 0.3$). These values show that in the case of frames with flexible beam-columns connections, the columns buckling critical loads are almost identical.

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