

PARAMETERS OPTIMIZATION OF SPRINGBACK IN SHEET METAL FORMING BY RESPONSE SURFACE METHOD

OPTIMISATION DES PARAMETRES DU RETOUR ELASTIQUE EN EMBOUTISSAGE DE TOLES PAR LA METHODE DE SURFACE DE REPOSE

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ABSTRACT

This paper deals with the optimization of Sheet Metal Forming parameters in order to reduce defect of springback which occur at the end of process. We propose a specific methodology based on the coupling between the one-step Inverse Approach as a surrogate model for the rapid simulation of sheet forming process, and a Response Surface Method based on diffuse approximation. The response surfaces are built using Moving Least Squares approximations constructed within a moving region of interest. Application of the sheet metal forming process is used to demonstrate the robustness of the method. The final design is validated with Stampack[®] and Abaqus[®] commercial codes based respectively on Explicit Dynamic and Implicit Static Approaches.

KEYWORDS: Sheet metal forming, inverse method, moving least square, springback.

RESUME

Cet article traite de l'optimisation des paramètres d'emboutissage de tôles afin de réduire le défaut du retour élastique qui se produit à la fin du procédé après retrait des outils. Nous proposons une méthodologie spécifique basée sur le couplage entre l'approche inverse pour la simulation rapide du procédé d'emboutissage de tôles, et la Méthode de surface de réponse basée sur l'approximation diffuse. Les surfaces de réponse sont obtenues en utilisant une approximation par moindre carrée mobile construites dans un domaine d'influence mobile. Une application du procédé d'emboutissage de tôles est utilisée pour démontrer la robustesse de la méthode. La forme finale est validée à l'aide des codes commerciaux Stampack[®] et Abaqus[®] basés sur les approches Explicit Dynamique et Implicite Statique respectivement.

KEYWORDS: Emboutissage, méthode inverse, moindre carrée mobile, retour élastique

1 INTRODUCTION

In this paper, the use of Moving Least Squares (MLS) based on Diffuse Approximation (DA) for the regression model is investigated along with strategies for progressive selection of points in the design space where designs are evaluated in the way to maximize the accuracy while minimizing the number of function evaluations. When the design of experiments and regression models are selected, any usual descent method can be used to find an optimum design. In the present work, we exploit further the MLS features and particularly the notion of pseudo-derivative to design a specific optimization technique, custom built for this regression model.

This new method is an extension of pattern search (Lewis and Torczon, 2002) in two aspects: (1) accommodation of arbitrary regular and irregular patterns; (2) design points eligible for inclusion in any pattern instance belong to a predefined set in the design space where experiments are authorized. This notion of authorized experiments limits the number of function evaluations and permits to explore large regions of interest by delaying an effective experiment to the moment when it is really needed. We note that without restriction (2), the approach becomes a generalization of the pattern search. By reducing further the size of the pattern towards small perturbations we obtain also a general, gradient-free descent method for a given regression fit. Gradient counterparts of all the three variants of the method

are obtained when using Hermite MLS development. We propose therefore a unified framework covering all the three domains: Response Surface Method (RSM), pattern search and general optimization for descent methods development, based on a common MLS regression model. In the final section of the paper, we present a numerical procedure combining a simplified finite element Inverse Approach (IA) as developed by (Batoz *et al.*, 1998) for the rapid stamping process modeling with our RSM.

2 OPTIMIZATION PROBLEM STATEMENT

In the optimization process, the goal is to

$$\text{minimize } f(\mathbf{x}), \mathbf{x} \in R^n \quad (1)$$

subject to a set of m constraints

$$g_j(\mathbf{x}) \leq 0, j = 1, \dots, m \quad (2)$$

$$\text{with } L_i \leq x_i \leq U_i, i = 1, \dots, n \quad (3)$$

where f is the cost function, x_i are the design variables, g_j is the j -th nonlinear constraint. The region of interest is defined by L_i and U_i which are respectively the lower and upper bounds on the design variables.

The RSM approach consists in solving a problem where the cost function (and eventually the constraint functions) are replaced by their approximations \tilde{f} and \tilde{g}_j . This simplified problem may be written as

$$\text{minimize } \tilde{f}(\mathbf{x}), \mathbf{x} \in R^n \quad (4)$$

subject to a set of m constraints

$$\tilde{g}_j(\mathbf{x}) \leq 0, j = 1, \dots, m \quad (5)$$

within the bounds **Erreur ! Source du renvoi introuvable.**

The approximations **Erreur ! Source du renvoi introuvable.** and **Erreur ! Source du renvoi introuvable.** are based on a set of numerical experiments with the function f . The problem of distributing the experimental points in the design space is known as Design of Experiment (DOE). The goal is to obtain a best accuracy with limited number of evaluations of f and g_j .

3 MOVING LEAST SQUARES MODEL

The approximate functions encountered in RSM rely mainly on multiple linear and second-order regression models, (Myers and Montgomery, 2002). The regression coefficients are fitted by least squares. The drawback is, that these models are global over a given region of interest. The idea applied by (Stander, 2001) is to make the

minimization algorithm progression by building new response surfaces centered on each successive minimum. During the progression of the process, the region of interest moves or zooms and new numerical experiments are performed at each iteration. However, the continuity between subsequent approximations is not guaranteed and the information from designs evaluated at previous iterations is difficult to take into account.

In the actual work, we explore the application of MLS techniques for the response estimation. The idea is to exploit the MLS capability to deal with irregular grids of data points accumulated during successive iterations. The approximation is local, what means that only the points closest to the current optimum are taken into account. The approximation coefficients are continuous when panning and zooming of the region of interest is performed. This allows for use of descent methods based on gradients of approximate response. A detailed presentation of MLS may be found in (Lancaster and Salkauskas, 1986) and is beyond the scope of the present paper. Here, we recall the matrix formulation of the approximation in an extent necessary to introduce our regression model.

Given the function values for a set of experimental points \mathbf{x}_i distributed according to a chosen DOE, the function \tilde{f} can be defined in terms of basis functions \mathbf{p} and some adjusting coefficients \mathbf{a} as

$$\tilde{f}(\mathbf{x}) = \mathbf{p}^T(\mathbf{x})\mathbf{a}(\mathbf{x}) \quad (6)$$

In absence of a specific knowledge about the behavior of the solution, a common choice for the basis functions \mathbf{p} are linear and quadratic monomials

$$\mathbf{p}(\mathbf{x}) = \left\langle 1 \quad x_1 \quad x_2 \quad \dots \quad x_n \quad x_1 x_2 \quad \dots \quad x_i x_{i+1} \quad \dots \quad \frac{x_1^2}{2} \quad \dots \quad \frac{x_n^2}{2} \right\rangle^T \quad (7)$$

For two variables the approximation is given by

$$\tilde{f}(x) = \left\langle 1 \quad x_1 \quad x_2 \quad \frac{x_1^2}{2} \quad x_1 x_2 \quad \frac{x_2^2}{2} \right\rangle \begin{Bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{Bmatrix} \quad (8)$$

The coefficients a_i are determined by a weighted least squares method minimizing the error $J(\mathbf{a})$ between the experimental and approximated values of the objective function

$$J(\mathbf{a}) = \sum_{i=1}^N w(\|\mathbf{x}_i - \mathbf{x}\|) (\mathbf{p}^T(\mathbf{x}_i - \mathbf{x})\mathbf{a} - f(\mathbf{x}_i))^2 \quad (9)$$

where N is the number of performed experiments and \mathbf{x}_i are the experimental designs. The weights w_i insure the

continuity and the locality of the approximation and are defined $w_i > 0$, decreasing within a fixed region around the point i called domain of influence of \mathbf{x}_i and vanish outside. The weight functions play a crucial role by influencing the way that the coefficients a_i depend on the location of the design point \mathbf{x} .

$$\text{Min}(J) \text{ gives } \mathbf{a}(\mathbf{x}) = \mathbf{A}^{-1} \mathbf{B} \mathbf{f} \quad (1)$$

$$\text{With } \begin{aligned} \mathbf{A} &= \mathbf{P} \mathbf{W} \mathbf{P}^T \\ \mathbf{B} &= \mathbf{P} \mathbf{W} \end{aligned} \quad (2)$$

$$\text{Where, } \mathbf{W} = \begin{bmatrix} w(\mathbf{x}_1 - \mathbf{x}) & & & 0 \\ & w(\mathbf{x}_2 - \mathbf{x}) & & \\ & & \ddots & \\ 0 & & & w(\mathbf{x}_n - \mathbf{x}) \end{bmatrix} \quad (3)$$

By construction, the MLS approximation represents exactly the basis functions p_i . Different versions of the approximation, like interpolating MLS may be obtained by a proper choice of the weights w_i .

The a_i may be interpreted as the coefficients of Taylor expansion of f around the evaluation point \mathbf{x} . In this sense, a_i correspond to the approximations of subsequent derivatives – we use the term of “diffuse derivative” and the symbol δ to differentiate from the “full” derivative ∂

$$\mathbf{a}(\mathbf{x}) = \left\langle \tilde{f} \quad \frac{\delta f}{\delta x_1} \quad \frac{\delta f}{\delta x_2} \quad \dots \quad \frac{\delta^2 f}{\delta x_n^2} \right\rangle^T \quad (13)$$

When the design sensitivity information is available, a Hermite version of MLS may be defined by introducing a supplementary term relative to gradient of f in the criterion **Erreur ! Source du renvoi introuvable.** yielding a modified criterion J_H

$$J_H(\mathbf{a}) = t \cdot J(\mathbf{a}) + (1-t) \sum_{i=1}^N r_i^2 w_i(\|\mathbf{x}_i - \mathbf{x}\|) (\nabla \mathbf{p}^T(\mathbf{x}_i - \mathbf{x}) \mathbf{a} - \nabla f(\mathbf{x}_i))^2 \quad (14)$$

where r_i is a characteristic size of domain of influence i , and the equilibrium between the two parts of the criterion is regulated by the coefficient $t \in [0,1]$.

In this formulation, the equations (1) and (2) remain valid with the matrices \mathbf{P} and \mathbf{W} (3) replaced respectively by \mathbf{P}_H and \mathbf{W}_H defined in the following manner

$$\begin{aligned} \mathbf{P}_H &= [\dots \quad \mathbf{p}_H(\mathbf{x}_i - \mathbf{x}) \quad \dots], \\ \mathbf{p}_H(\mathbf{x}) &= \begin{bmatrix} \mathbf{p} & \frac{\partial \mathbf{p}}{\partial x_1} & \dots & \frac{\partial \mathbf{p}}{\partial x_n} \end{bmatrix} \end{aligned} \quad (13)$$

The formulation (14) reduces to **Erreur ! Source du renvoi introuvable.** when $t = 0$. Sensitivity information is crucial when high number of design variables is considered, so it should be used whenever possible. The Hermite MLS formulation accommodates easily such cases when only a subset of gradients is available.

$$\begin{aligned} \mathbf{W}_H &= \begin{bmatrix} w_H(x_1) & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & w_H(x_n) \end{bmatrix}; \\ \mathbf{w}_H(\mathbf{x}_i) &= \begin{bmatrix} t w(\mathbf{x}_i) & 0 \\ 0 & r^2(1-t) w(\mathbf{x}_i) \mathbf{I}_{n \times n} \end{bmatrix} \end{aligned} \quad (14)$$

4 DESIGN OF EXPERIMENTS (DOE)

The successive response surface method uses a set of points of the design space to determine an approximate optimum. A new region of interest centers on each successive minimum and its size is determined by choosing a range for each variable. This set of points in the region of interest for which we perform numerical experiments is an instance of what is called a pattern, from which pattern search approach takes its name. Progress is made by panning the pattern and eventually zooming at its center. **Erreur ! Source du renvoi introuvable.** shows the possible evolution of a 4-point pattern in 2D between iterations i and $i+1$. At the iteration i , the point \mathbf{x}_i forms the center of the current working region of interest.

The lower and upper bounds of this sub region are denoted in the figure by thick black dots and form the current pattern. The cost function f and eventually the constraints g_i are calculated for these points. The translation of the pattern for the next iteration is given by a descent method applied to the response surface fitted on black dot points. The evolution of the patterns size depends on the nature of the solution and on the accuracy of the current optimum. In **Erreur ! Source du renvoi introuvable.**, the pattern taken into account at the iteration $i+1$ is given by the gray dots. However, a straightforward implementation of this approach does not allow to take into account at iteration $i+1$ the designs computed at iteration i , even when they belong to the new region of interest. This is the case of the upper-right black dot in **Erreur ! Source du renvoi introuvable.**

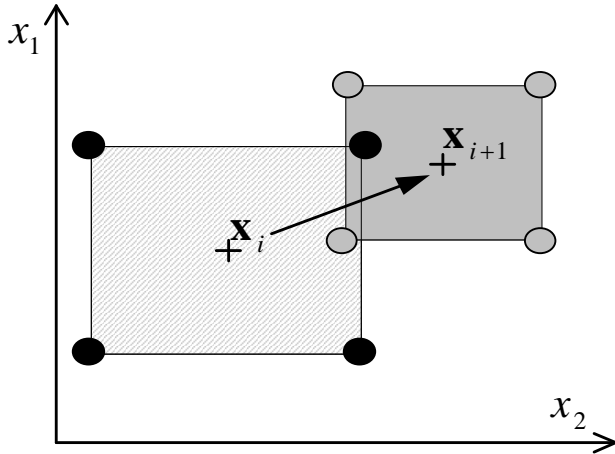


Figure 1: Pattern search with panning and zooming of the region of interest

In **Erreur ! Source du renvoi introuvable.** the hollow dots show a discrete set of points in design space where we decide to authorize the numerical experiments. This is what we call a virtual DOE as the experiments are designed but not yet performed at this stage.

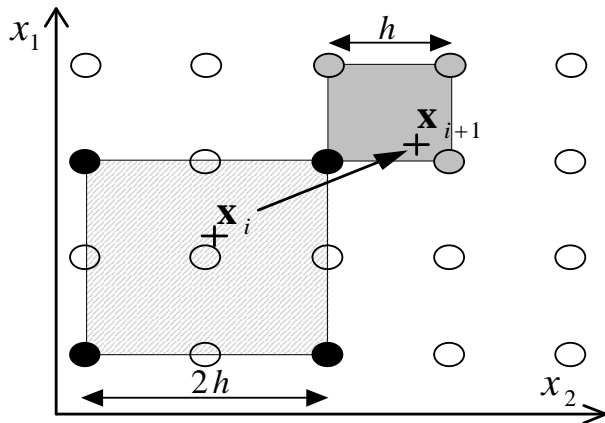


Figure 2: Multiscale pattern search with Virtual DOE

The idea is, that a virtual design is evaluated only when it enters the actual region of interest. In **Erreur ! Source du**

renvoi introuvable. the point \mathbf{x}_i was centered within the region of interest. In the actual approach, the search pattern

is no more centered on \mathbf{x}_i but is defined by the set of closest virtual designs and the scale of the resolution of the grid. In **Erreur ! Source du renvoi introuvable.** at iteration i the pattern is defined by four solid black dots for resolution window size $2h$. The MLS response surface is then fitted on these points. When the current design is translated to \mathbf{x}_{i+1} , the closest neighbors are selected with

resolution refined to h . The new points are denoted by the three gray dots and the fourth one is reused from the previous pattern. In this way, the total number of experiments is reduced from 8 to 7. The gain is proportionally higher for bigger patterns.

Moreover, taken into account the nonlinear nature of MLS, several iterations may be performed within a fixed pattern as long as the list of closest neighbors does not change, further reducing the number of experiments.

5 OPTIMIZATION ALGORITHM

The goal of the iterative optimization process is to reach a minimum point for the objective function. An iteration of the optimization algorithm is organized as follows:

- advance the current design point \mathbf{x}
- search for the function values f_i in the neighborhood of \mathbf{x}
- update the MLS coefficients a_i using (1)
- compute the next move $\Delta\mathbf{x}$

This algorithm is valid for both the symmetric and the asymmetric patterns. The difference between the two variants lies in the interpretation of the step 2. For a symmetric pattern, new data points x_i are generated and evaluated at each iteration. In the asymmetric case, the design points are allowed on a predefined grid and evaluated values are gradually recorded in a database. The step 2 consists then in selecting the set of closest points, retrieving the computed values from the database and computing the missing ones. The database is then updated. There are several details to be considered when implementing the algorithm. Some of them, of technical character. In this category we cite the relaxation coefficients used to limit the move size in order to avoid premature convergence. Other are still subject of active research. This concern the computation of the next move in step 4 is more complicated in presence of constraint functions.

6 THE SPLITTING RING BENCHMARK

6.1 Problem description

In this section we will present the study of an interesting springback problem proposed and studied experimentally by M.Y. Demeri from Ford[®] Laboratory (Demeri *et al.*, 2000). It consists in three operations: First a classical deep drawing operation is carried to get a cylindrical cup (Figure 3.a), then the cup is cut to get a circular ring specimen (Figure 3.c), and the third operation is a radial splitting to open the ring under the initial stress effect (Figure 3.d).

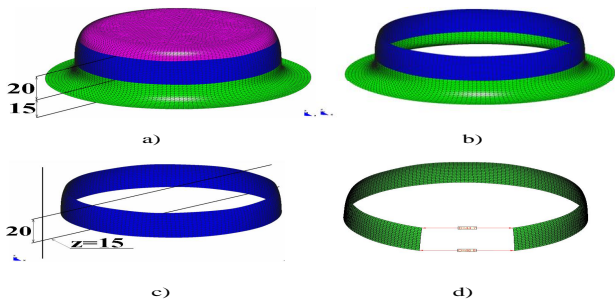


Figure 3: Splitting Ring Benchmark – Main operations

The objective of this application is to simulate the forming and the springback processes to evaluate the ring opening gap value after springback.

6.2 Deep drawing simulation

The Geometric and material data are given in figure 4. The fast Inverse Approach (one-step method) has been used to carry out the deep drawing simulation using the commercial Stampack-OneStep[®] code (Quantech ATZ, 2003). Only a quarter of the cup is modeled by 3150 DKT12 (Batoz *et al.* 1998) shell elements (Figure 5) with 5 Lobatto points through the thickness for plasticity integration and 6 degrees of freedom per element. The obtained results have been compared to those obtained for the same problem but using both two incremental methods: The Implicit-Static using Abaqus[®] Standard code with the S3R triangular shell element and the Explicit-Dynamic using Stampack-Incremental[®] commercial code with the BST rotation-free triangular shell element.

In order to validate the results obtained using the one-step method, we compare bending moments and stresses at the end of forming between different codes. Figure 6 shows bending moment distribution M_{xx} on the final drawn cup. We can observe a good results correlation between the two methods.

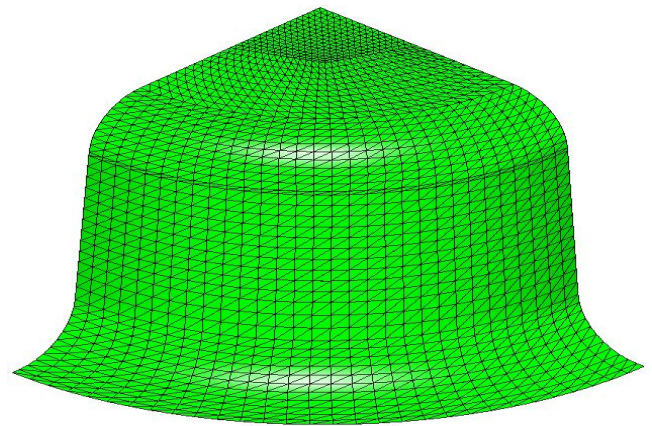


Figure 5: Mesh of the 1/4 of cup, 3150 DKT12 shell elements

From the qualitatively point of view, we represent circumferential stress distribution on the top flange along the curvilinear abscissa in figure 7. The same conclusion can be drawn; also we can remark good agreement of the results between the two methods.

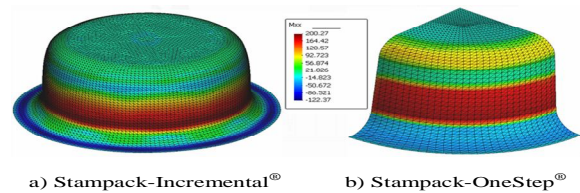


Figure 6: Bending moment distribution- comparison

Because of its quite good quality results and its fewer CPU consuming time (see Table 1), the one-step method has been chosen for the deep drawing simulation problem coupled with the MLS method for the optimization of tools geometry (section 6.4).

Thus, we can carry out the springback simulation using the final stresses obtained using the one-step method.

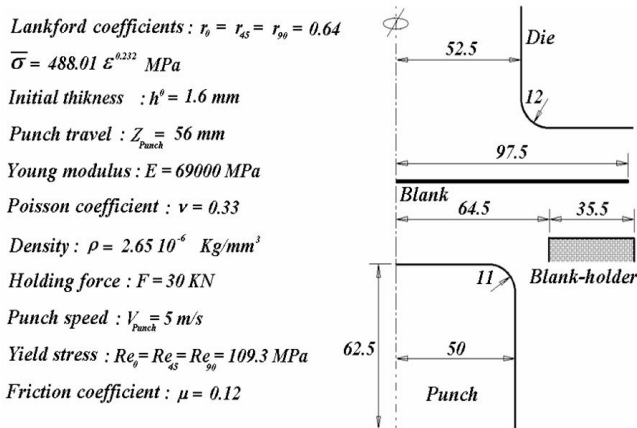


Figure 4: Geometrical and material data

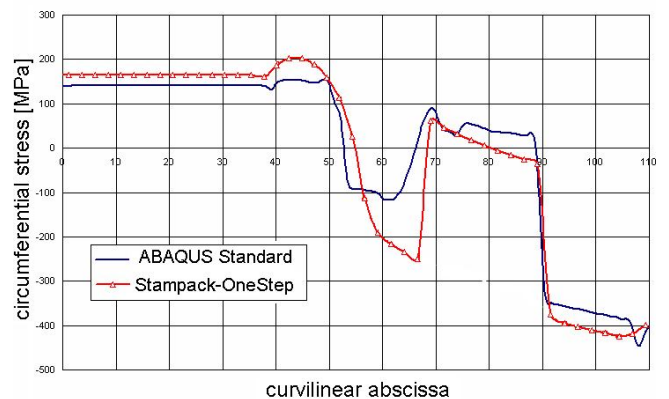


Figure 7: Stress distribution on the top flange- comparison

6.3 Springback simulation

When using the one-step method, the simulation of the springback process is done without modeling tools. Thus, the internal efforts (membrane forces and bending moments) obtained at the end of forming stage, are taken to be as initial efforts for the springback simulation. In Stampack-OneStep[®] code the springback simulation is done based on the fast incremental Updated Lagrangian Formulation (ULF). One half of the ring specimen is modeled using 1200 DKT12 shell elements with 12 degrees of freedom per element. The half of ring is clamped on one side and free on the other (Figure 8).

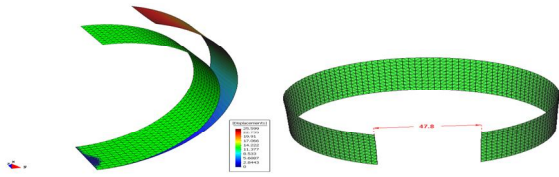


Figure 8: Ring opening after springback

The fast springback simulation was carried using ten steps, the principal springback results compared to others obtained using different codes are summarized in table 1. As we can see, results obtained with the one-step method are in good agreement with those obtained by Abaqus[®] and the experiments. We can also note that, CPU consuming time for the one-step method is insignificant compared to those of other methods.

6.4 Die radius optimization

The objective of the optimization procedure is to find the best die radius which leads to a minimum gap after springback. Figure 9 presents the definition of design variables and the objective function to minimize which is given by:

$$J = \sqrt{\sum_{i=1}^{nmt} (\vec{d}_i^T \cdot \vec{d}_i)} = \sqrt{\sum_{i=1}^{nmt} (u_{Xi}^2 + u_{Yi}^2 + u_{Zi}^2)} \quad (17)$$

Table 1: Summary of principal springback results - comparison

Method	nb. elem.	Gap [mm]	CPU time
Stampack- Incremental [®]	7000	45.7	2h 33m 53s
Abaqus [®] Standard	6600	48.6	7h 09m 48s
Gati et al. 2001	9984	45.0	2h 05m 20s
Experiments		50	
Stampack-OneStep [®]	3150	47.8	0h 00m13 s

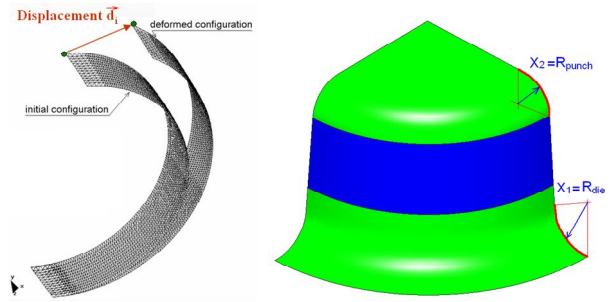


Figure 9: Definition of the optimization problem

The design variables (punch and die radii) are subjected to bounds constraints defined by:

$$6mm \leq x_i \leq 18mm \quad (18)$$

The initial value of the design variable is $x=12$ mm. The real objective function is evaluated at each point during optimization process. The achieved minimum inside the working domain corresponds to $x= x_{max}=18$ mm. The reduction of the objective function is given in Figure 10.

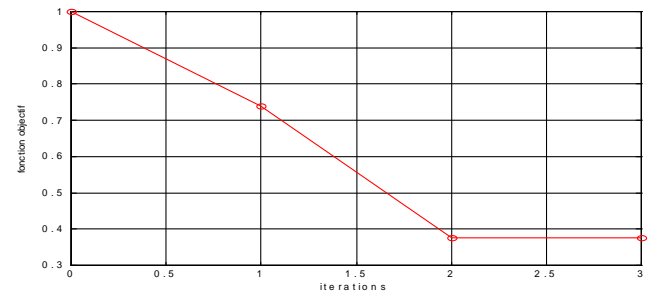


Figure 10: Reduction of the objective function

We can observe a good convergence property, only four iterations were necessary.

The opening gap was calculated at the end of the optimization procedure, the value after optimization is about 21.3 mm, which is smaller than the initial value of 47.8 mm.

6.5 Material parameters identification

The second optimization problem deals with material parameters. The objective is to determine the best material parameters to reduce the ring opening gap. The design variables are the two material hardening parameters K and n . Optimization variables are constrained between $200 \leq K \leq 600$ MPa and $0.25 \leq n \leq 0.55$. A regular design of experiment is used based on 7×5 points where the objective function is evaluated initially by a sampling procedure before the optimization process.

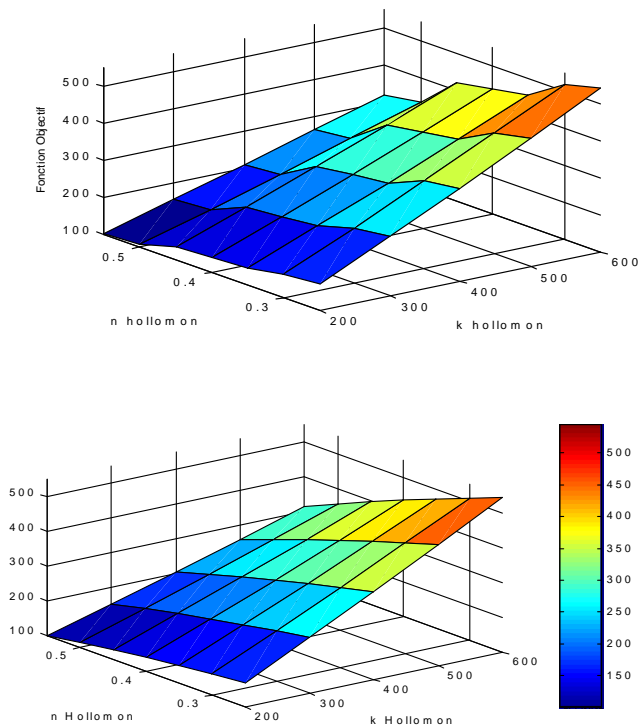


Figure 11: Response surface approximation

The pattern search used is based on a TCR9 with 9 points. Figure 11 shows the real function sampling and the approximated response surface. We can observe a good agreement between the two surfaces. The approximated response surface is optimized using the MLS method, the global minimum corresponding to $(K = 200, n = 0.55)$ was obtained.

7 CONCLUSION

We proposed a new response surface method involving Moving Least Squares regression models and pattern search optimization. The resulting response surface algorithms involve iterative improvement of the objective and constraint functions employing locally supported nonlinear approximations. We explored the strategies to reuse the experimental designs of previous iterations in order to reduce the number of costly finite element simulations.

In the metal forming design example we built the response surfaces using a surrogate model based on a simplified one-step Inverse Approach analysis. The resulting procedure was applied successfully for the design of tools geometry of ring benchmark. The validation of results using Stampack[®] explicit industrial code and Abaqus[®] implicit code showed that the proposed procedure used along with Inverse Approach surrogate model is well suited for deep drawing process design. These new methods needs to be further tested on larger examples with more design variables and nonlinear constraints.

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